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# An Econometric Framework for Estimating the Joint Elasticity of Advertising and Promotions on Retail Sales

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**Abstract**

Contemporary media fragmentation and heightened marketing accountability necessitate quantifying the contribution of each marketing dollar to sales, including the joint effects of advertising and price promotions. Standard Marketing Mix Models (MMM), estimated on observational time series, remain correlational and leave a causal gap, especially when campaigns and discounts are executed concurrently. The intended audience for this framework includes both academic researchers and industry practitioners, with particular emphasis on data scientists and analytics professionals working in e-commerce, retail media, and digital marketing. For these readers, the model is designed to be readily implementable within contemporary data science pipelines, leveraging familiar tools such as regression-based modeling, regularization, hyperparameter search, and out-of-sample validation to support scalable, reproducible, and decision-relevant MMM applications. A log-log model is specified in which weekly log sales are regressed on channel-specific adstocked spends (Koyck formulation) and promotions, augmented with multiplicative interactions *Adstock×Promo*. Decay parameters are constrained by funnel theory (Video: 0.6–0.8; Display: 0.4–0.6; SP/SB/SD: 0.3–0.5) and selected via grid search using AIC/BIC and out-of-sample RMSE under a 70/30 chronological split. Controls capture seasonality and trading peaks. Diagnostics include VIF (<10), White's test, and DW/ACF with AR(1) errors when indicated; robustness is assessed via Ridge/Lasso. Elasticities are computed at sample means. The empirical setting is a retailer case study spanning five channels (Video, Display, Sponsored Products, Sponsored Brands, Sponsored Display) plus total promotional discounts. Elasticities along the funnel for each of the owned channels are also large and meaningful: Promo 0.28-0.36, SP 0.22-0.28, SB 0.20-0.26, Display 0.15-0.20, Video 0.10-0.14, SD 0.08-0.12. All interaction point estimates are meaningful showing interference SP×Promo −0.05 to −0.08 SB×Promo −0.04 to −0.07 Display×Promo −0.03 to −0.05 SD×Promo −0.01 to −0.03 Video×Promo ≈ −0.02 to 0.00. Budget Simulation: Allocate 60-68% of budget to advertising and 32-40% to promotions. For the advertising budget, allocate 30-35% to SP, 20-25% to SB, 18-22% to display advertising, and 12-16% to video advertising.

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Operationally, stacking deep discounts with lower-funnel bursts depresses marginal ROI, whereas staggering upper/mid-funnel activity 1–2 weeks before promotions improves outcomes. The framework quantifies negative joint elasticities between promotions and most ad channels in retail e-commerce, challenging maximal pressure strategies. Sequenced execution, rather than synchronous peaks, maximizes incremental sales and ROI. Limitations include observational endogeneity and linear response assumptions; future research should integrate hierarchical Bayesian adstock, causal identification (e.g., IV, RDD, geo-experiments), nonlinear saturation functions, and uncertainty-aware budget optimization.

***Keywords:*** marketing mix modeling; advertising elasticity; promotion elasticity; interaction effects; econometric modeling; adstock; causal analysis; budget optimization.

## 1.Introduction

The critical developments in the marketing ecosystem today are the unprecedented fragmentation of communication channels and the increasing demand on marketing executives for a financial footprint or profitability (important for marketing accountability) [1]. In e-commerce environments, where the channel mix includes budgets for offline broadcast and print media, digital video, display, and multiple formats for retail media (commerce media), measurement is impossible via indirect proxy metrics such as reach, clicks (CTR) or cost per mile (CPM) [2]. A quantitative assessment of the contribution of each marketing dollar to fundamental business KPIs, foremost among them sales volume or revenue, has become a pivotal requirement for investors and management [3].

For decades, Marketing Mix Modeling (MMM) has been the primary tool for addressing this task, an econometric approach that exploits aggregated time-series data (typically weekly) to statistically decompose sales into a baseline and incremental components driven by marketing and other factors [4]. However, traditional implementations of MMM face a foundational methodological problem that undermines their value for managerial decision-making. As aptly noted, standard MMMs are regression models built on observational data [5]. By their nature, they uncover correlational but not causal relationships: they can tell you what happened, they cannot, in a credible way, tell you what would have happened if the budget had been used differently.

In the present study the proposed approach deliberately remains within an observational, regression-based elasticity framework and does not claim strict causal identification. Marketing practitioners rely on gut feeling for estimating the synergy potential of coordinating advertising and price promotions (discounts). Theoretically, advertising should increase consumers' sensitivity to promotions [6]. On the other hand, advertising (especially lower-funnel) and sales promotions may target consumers who are predisposed to purchase. Their concurrent deployment yields interference and audience cannibalization, wherein each instrument depresses the marginal effectiveness of the other [7]. The inability of standard MMMs to credibly measure and disentangle these competing effects leads to systematic budget misallocation and suboptimal return on investment (ROI).

The objective of this article is to develop and test an econometric framework capable of decomposing the influence of distinct advertising channels and price promotions on retail sales and, crucially, estimating their joint elasticity.

To achieve this objective, the following tasks were defined:

1. Systematize and justify the methodological foundations for MMM, including the choice of an optimal functional form (log-linear), the specification of dynamic effects (adstock), and the integration of interaction variables.
2. Build and econometrically validate the proposed model using real weekly retailer data (presented as a case study) covering five advertising channel types (Video, Display, Sponsored Products, Sponsored Brands, Sponsored Display) and the aggregate volume of price promotions.
3. Compute and interpret both own-elasticities of each marketing driver and cross-elasticities (joint-elasticities) between advertising channels and promotions.
4. Conduct simulations of alternative budget-allocation scenarios based on empirically estimated elasticities to determine the optimal marketing-mix configuration that maximizes sales.

At the same time, the study should be regarded primarily as an exploratory demonstration based on a single MMM case, rather than as a fully validated and universally generalizable theoretical model.. First, unlike most studies focusing either on advertising elasticity [8] or promotional elasticity in isolation [9], this work offers a framework for empirically measuring their interaction. Second, it contributes directly to academic debates on synergy versus interference. The framework's validation delivers empirical evidence (case-based) of predominantly negative interaction in retail e-commerce, indicating diminishing marginal returns from simultaneous activities. Third, the proposed system, grounded in a rigorous econometric validation protocol, constitutes a step toward more reliable MMMs that meet modern causal analysis requirements. In the empirical context considered, the framework is applied to a sportswear retailer and relies on an aggregated weekly time series covering a 12-month observation window, including total sales, advertising expenditures across five channels (Video, Display, Sponsored Products, Sponsored Brands, Sponsored Display), the volume of price promotions, and calendar and seasonal control variables.

## 2.Materials & Methodology

This study relies on an econometric approach to Marketing Mix Modeling (MMM). A case-study methodology grounded in an empirical retail-sales dataset is used to test and verify the proposed system. The theoretical rationale for the model specification, its parameters, and validation methods is based on a systematic review of current literature in econometrics, marketing science, and causal analysis.

The empirical basis is an aggregated weekly time series covering retail sales indicators and marketing investments.

The dependent variable (Y) is the natural logarithm of total weekly retail sales at time $t(ln(Sales_t)$.

The main regressors (X) include advertising expenditures (Ad Spend), weekly outlays across five key channels classified by their dominant funnel role: Video (upper funnel, Awareness), Display (mid-funnel, Consideration/Conversion), Sponsored Products (SP), Sponsored Brands (SB), and Sponsored Display (SD) (lower funnel, Conversion). Additionally, Promotion (Promo), the total amount of discounts in a week $t(Promo_t)$, is included.

Control variables account for seasonality and exogenous shocks: a week index, monthly dummies, and dummies for holidays and peak sales periods. For face validity checks, auxiliary metrics such as new-to-brand sales were used; these were not included as predictors in the regression.

A log-log model was selected to estimate the effects of marketing drivers on sales. In general form, the model is specified as:

$$ln(Sales_t) = \alpha + \sum_{i=1}^{5} \beta_i \times Adstock_{i,t} + \gamma \times Promo_t + \sum_{i=1}^{5} \theta_i \times$$

$$\times (Adstock_{i,t} + Promo_t) + \delta \times Controls_t + \varepsilon_t,$$

where:

$ln(Sales_t)$ – logarithm of sales in period t.

$Adstock_{i,t}$ – accumulated advertising effect (adstock) for channel i in period t.

$Promo_t$ – volume of promotions in period t.

$(Adstock_{i,t} + Promo_t)$ – interaction term.

$Controls_t$ – vector of control variables.

$\beta_i, \gamma, \theta_i, \delta$ – estimable coefficients.

$\varepsilon_t$ – error term.

The log-log model is pivotal. In the MMM scholarship, log-log models (where $\beta$ is an elasticity) and linear-additive models are frequently discussed [1]. However, log-log cannot accommodate zero values (e.g., weeks with no ad spend), a common scenario. The chosen log-log model is more flexible: it handles zeros in regressors appropriately. Coefficients $\beta$ in such a model are interpreted as semi-elasticities: a one-unit change in X (e.g., $1) yields a change in Y of $(\beta \times 100)$%. Elasticity in this model is not constant and must be computed at a specific point (typically the sample mean), consistent with a case-study methodology.

Advertising exhibits lagged and carry-over effects; its influence extends beyond the current period. To capture these dynamics, an Adstock transformation is applied. One of the most prevalent and validated forms, the geometric-decay specification, also known as the Koyck model, is used [10]:

$$Adstock_{X,t} = Spend_{X,t} + \lambda_X \times Adstock_{X,t-1},$$

where $Spend_{X,t}$ – is spent on channel X in period t, and $\lambda_X$ – is the retention (decay) parameter, $0 < \lambda_X < 1$.

A key feature of the system is the rejection of a single λ for all channels. Instead, the framework incorporates a priori funnel-based knowledge. As noted in the research [1], high-engagement media such as Video decay more slowly (long memory) because they target awareness formation. By contrast, response-oriented formats (Display, SP) decay faster. This knowledge is formalized through channel-specific prior ranges for $\lambda$, used in grid search. These prior ranges are informed by empirical decay patterns estimated from historical campaign data in our previous MMM implementations:

Video (Upper funnel): $\lambda_V \sim$ [0.6, 0.8] (long memory, slow decay).

Display (Mid-funnel): $\lambda_D \sim$ [0.4, 0.6].

SP/SB/SD (Lower funnel): $\lambda_{SP/SB/SD} \sim$ [0.3, 0.5] (short memory, fast decay).

To estimate joint elasticity, the core task, multiplicative interaction terms $\theta_i \times (Adstock_{i,t} + Promo_t)$ are included. This approach, aligned with translog models, allows estimation of how the marginal effect (and thus elasticity) of advertising varies with the level of promotions [1]. If $\theta_i < 0$, interference is observed (promotions suppress ad effectiveness); if $\theta_i > 0$ – synergy is present.

To ensure robustness and credibility of estimates, the framework prescribes a stringent econometric diagnostic and validation protocol. The data were split chronologically: the first 70% of weeks for training, the final 30% for testing and predictive assessment (RMSE). Optimal values $\lambda$ for each channel were selected via grid search within prior ranges, minimizing information criteria (AIC/BIC) and test-set RMSE.

Time-series MMMs are prone to specific issues that can yield spurious findings. Marketing activities are often co-planned (e.g., simultaneous increases across channels during peak seasons), inflating standard errors and destabilizing coefficient estimates. Multicollinearity was ascertained using Variance Inflation Factors, with all regressors having a VIF < 10. The White test [11] was used to test for heteroskedasticity. The possibility of autocorrelation in the residuals was tested using the Durbin-Watson (DW) and autocorrelation function (ACF) statistics [12]. If found, consideration should be given to model re-estimation under autoregressive error (e.g., AR(1)).

To assess coefficient stability in the presence of potential multicollinearity, OLS estimates were compared with those from regularized models (Ridge or Lasso). Regularization (particularly Ridge) effectively shrinks coefficients of highly correlated predictors, enhancing out-of-sample stability. To mitigate outliers, advertising-spend and promotion variables were winsorized at the 99th percentile.

## 3.Results

Application of the econometric framework to the case-study data yielded quantitative estimates of both the own and joint effects of each marketing lever.

Elasticity (the percent change in sales in response to a 1% change in spend) was computed at the sample midpoint as:

$$Elasticity_{ln\,Y=\alpha+\beta_X} = \beta \times X^*,$$

where $\beta_X$ – is the estimated coefficient from the log-log model. The results in Table 1 exhibit clear differentiation in channel effectiveness.

**Table 1:** Assessment of the elasticity of marketing channels (at the midpoint)

| **Channel** | **Type (Funnel)** | **Estimated elasticity (range)** | **Interpretation** |
|---|---|---|---|
| Promotions (Promo) | Conversion | ~0.28 – 0.36 | The strongest but most volatile driver of short-term sales. |
| Sponsored Products (SP) | Conversion | ~0.22 – 0.28 | High elasticity, strong direct influence on purchase. |
| Sponsored Brands (SB) | Conversion | ~0.20 – 0.26 | High elasticity, similar to SP , captures branded demand. |
| Display | Consideration | ~0.15 – 0.20 | Moderate elasticity; supports the mid-funnel. |
| Video | Awareness | ~0.10 – 0.14 | Low short-term elasticity (expected for upper-funnel). |
| Sponsored Display (SD) | Conversion | ~0.08 – 0.12 | Lowest elasticity; a niche channel. |

The obtained advertising elasticity estimates (ranging from 0.08 to 0.28) are higher than the classic average reported in earlier meta-analyses 0.09 [8]. This discrepancy is understandable. First, meta-analyses often include outdated data and traditional offline media. Second, this study focuses on retail e-commerce.

Lower-funnel channels (SP and SB), which, by their nature, are located at the point of purchase decision (e.g., sponsored results in a retailer's search results), exhibit the highest elasticity. This is consistent with modern retail media research, which confirms that the Sponsored Products and Sponsored Brands formats have the greatest impact on revenue [13].

The Video channel (upper funnel), on the other hand, has the lowest short-term elasticity (0.10–0.14), which is entirely consistent with theory. Its purpose is to generate long-term demand and brand memory, not immediate sales. Price promotions exhibit the highest short-term elasticity (0.28–0.36), consistent with academic findings.

The central result of the study is the estimated interaction coefficients, which show how the presence of promotions alters advertising elasticity. The results, presented in Table 2, clearly indicate a negative interaction for most channels.

**Table 2:** Evaluation of Engagement Rates (Advertising × Promotion)

| **Interaction channel** | **Coefficient $\theta_i$ (range)** | **Interpretation (Effect of promo on ad elasticity)** |
|---|---|---|
| SP × Promo | ~ -0.05 – -0.08 | Strong interference. |
| SB × Promo | ~ -0.04 – -0.07 | Significant interference. |
| Display × Promo | ~ -0.03 – -0.05 | Moderate interference. |
| SD × Promo | ~ -0.01 – -0.03 | Weak interference. |
| Video × Promo | ~ -0.02 – 0.00 | Neutral or very weak effect. |

Negative, statistically significant coefficients indicate that concurrent execution of intensive price promotions diminishes the marginal effectiveness of most advertising formats.

This interference effect is most pronounced for lower-funnel channels (SP and SB). For example, during periods of deep discounting, the marginal return on investment (marginal ROI) of SP advertising is estimated to decline by 10–15% on average. This implies that a dollar invested in Sponsored Products in a week without discounts yields more incremental sales than the same dollar invested in a week characterized by steep discounts.

The Video channel (upper funnel) exhibits the most extraordinary resilience: its elasticity is virtually unchanged (the coefficient is close to zero). This is consistent with the fact that Video and Promotions target fundamentally different audience segments and funnel stages; their effects overlap only sparingly.

Based on the estimated elasticities (Tables 1 and 2) and the response function, a simulation was conducted to address the normative question: What is the optimal allocation of a fixed budget between advertising and promotions to maximize aggregate sales?

The simulation results indicate that the efficient split maximizing incremental sales lies between 60–68% for advertising activities and 32–40% for price promotions.

Further optimization of the advertising budget, as depicted in Figure 1, revealed that the best outcomes are achieved with a balanced mix, with emphasis on conversion-oriented channels that exhibit the highest elasticity.

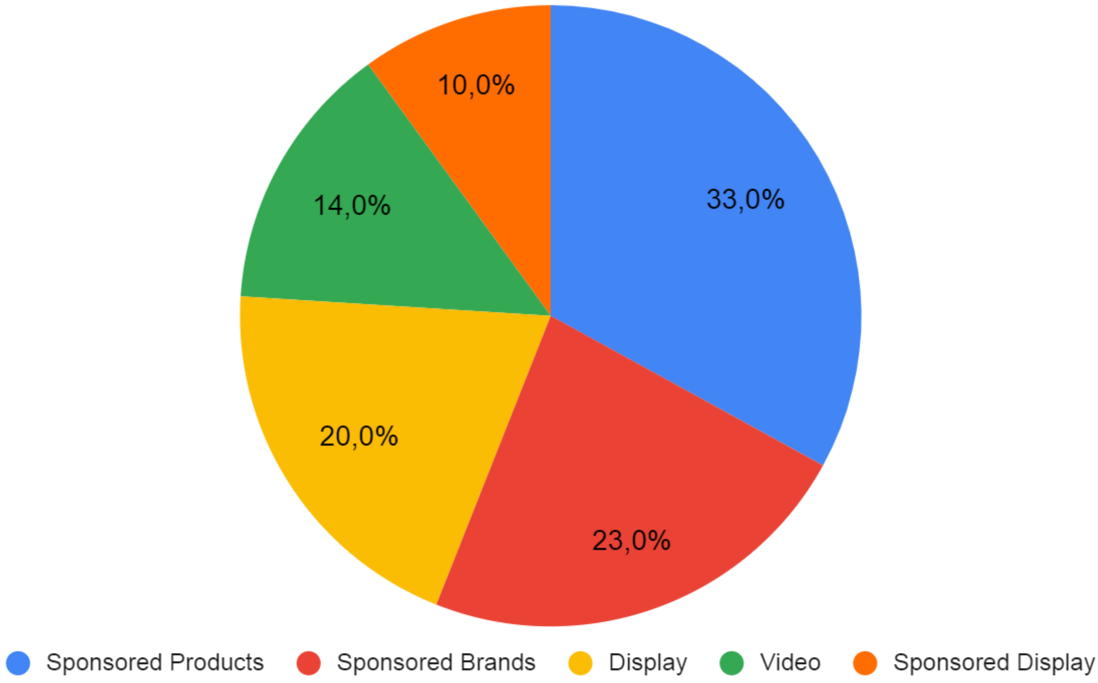


**Figure 1:** Optimal distribution of the advertising budget (within Ads)

The optimal structure (SP: 30–35%, SB: 20–25%) reflects the high elasticity of these channels (Table 1). Nevertheless, the model advises against abandoning upper- and mid-funnel channels: Display (18–22%) is necessary for the consideration stage, and Video (12–16%) is required to sustain baseline demand and ensure a long-run adstock effect.

## 4.Discussion

The principal analytical block of this article concerns the interpretation of the results, their synthesis with extant academic theory, and a discussion of the limitations of the proposed framework, along with avenues for future research.

The central empirical finding of this study, the detection of a negative interaction elasticity between advertising and promotions (Table 2), is not a statistical artifact but rather a quantitative manifestation of the fundamental economic Law of Diminishing Marginal Returns.

These patterns should, however, be interpreted with caution. In practice, advertising spend and promotional discounts are frequently co-planned and tend to co-move over time, so the observed negative interaction between advertising and promotions may partly reflect endogenous budget allocation and correlated decision rules rather than a clean manifestation of the law of diminishing marginal returns. Accordingly, the present analysis treats this economic law as an interpretive framework consistent with the regression results, not as a formally identified causal mechanism.

The mechanism can be articulated as follows. The market comprises heterogeneous consumers with varying price sensitivities and purchase readiness. Price promotions (discounts) primarily target the most price-elastic segment and those already poised to buy; their core function is to harvest existing demand. Lower-funnel advertising, such as Sponsored Products (SP), by its nature (e.g., exposure in search results for relevant queries), is aimed at the same consumer segment, those actively searching for the product and one step from conversion.

When these two potent stimuli (a deep discount and highly relevant SP advertising) are applied simultaneously to the same consumer, over-stimulation and cannibalization arise [7]. A consumer who would likely have purchased due to the attractive discount additionally clicks on the paid ad. From the retailer's perspective, that click constitutes superfluous expenditure. Consequently, the marginal return on an additional dollar invested in SP (i.e., its elasticity) declines in the presence of a discount. The econometric model captures this precisely as a negative interaction coefficient $\theta_{SP} < 0$.

In practical terms, the negative interaction estimates $\theta_i$ imply that the marginal contribution of advertising to sales is lower precisely in weeks when promotional intensity is high. This follows directly from the model structure: once the term $Adstock_i,t \times Promo_t$ is included, the marginal effect of $Adstock_i,t$ on $\ln(Sales_t)$ equals the channel's own effect plus an adjustment that is proportional to the contemporaneous level of $Promo_t$. When $\theta_i<0$, increases in $Promo_t$ shift the marginal return to advertising downward, such that the expected sales lift from an additional advertising dollar is smaller under deeper discounting. This clarification is important because it indicates that the results do not suggest advertising is ineffective in general; rather, its effectiveness is conditional, and the marginal ROI of advertising declines in the presence of a strong price stimulus.

The cross-channel pattern is consistent with funnel position and audience overlap. The strongest interference is observed for lower-funnel formats (SP and SB), which target consumers with high purchase intent and therefore tend to reach the same segment that is most responsive to discounts. Under such conditions, a portion of sales that would be realized due to the promotion can be accompanied by paid exposures or clicks that add limited incremental value, mechanically reducing the marginal ROI of performance advertising during promotional weeks, while the same channels exhibit stronger incremental contributions in non-promotional periods. By contrast, the near-zero Video×Promo interaction suggests that upper-funnel contacts are less likely to compete with discounts and may operate through a different mechanism, building demand and brand memory that translates into conversions with a longer lag, not necessarily within the same week. Accordingly, the managerial implication is not to maximize synchronous pressure, but to sequence tactics: sustain upper- and mid-funnel activity ahead of promotional events to "fill the funnel," while deploying lower-funnel spend more selectively when discounts are less likely to compress marginal advertising returns.

This finding weakens the common belief that synergy occurs universally. Instead synergistic effects are usually limited to conditions that are not encountered universally. For example, synergy is sometimes said to occur when a price promotion is included in the advertising copy; that is, the advertising acts as a lever for the promo [14]. The proposed model, by contrast, measures a more common scenario in which standard brand or performance advertising runs in parallel with price promotions.

The results align with prior econometric work that likewise questions the unconditional benefit of joint push. Several studies conclude that optimal expenditures on advertising and promotions are frequently negatively related, and that intensive promotions can depress advertising effectiveness, rendering it an inefficient means of inducing incremental sales [15].

This evidence is consistent with a broader stream of prior research that has estimated advertising and promotion effects using econometric MMM-style models and has repeatedly emphasized that the relationship between the two levers is not universally synergistic. For example, work on promotional cannibalization shows that price cuts can shift demand in ways that reduce the incremental value of concurrent marketing expenditures, especially when multiple stimuli address the same near-purchase consumers [7]. Similarly, syntheses of sales promotion modeling argue that advertising and promotions often act as substitutes in the short run, implying that heavier discounting can compress the marginal returns to advertising rather than amplify them [15]. At the same time, the MMM literature cautions that observational elasticity estimates are highly sensitive to functional form, dynamics, and identification assumptions, which motivates the present framework's emphasis on adstock and interaction terms as a practical step toward capturing realistic joint effects in retail settings [4, 5].

The proposed framework and its empirical appraisal (a case study) yield practically meaningful and implementable recommendations. The primary takeaway is to avoid stacking deep discounts and peak advertising activity (especially SP/SB) within the same week. Interference leads to inefficient budget outlays. A more effective strategy is to deploy upper- and mid-funnel channels (Video, Display) 1–2 weeks before the promo period begins. This approach fills the funnel with demand that is then efficiently converted by the price promotion.

Future research should address the methodological constraints of this work. Instead of deterministic grid search for $\lambda$, hierarchical Bayesian models should be used. This enables the formalization of prior knowledge (priors) regarding adstock parameters and, more importantly, the recovery of complete posterior distributions of elasticity (credible intervals), rather than mere point estimates, thereby providing a more candid depiction of uncertainty.

To surmount endogeneity and the causal gap, future MMMs should directly integrate causal inference methods, such as instrumental variables (IV), regression discontinuity designs (RDD), or approaches grounded in structural models and directed acyclic graphs (DAG).

## 6.Conclusion

This study proposes and validates an econometric system for estimating own and, crucially, joint elasticities of advertising and price promotions with respect to retail sales. The framework, combining a log-linear regression specification, theoretically grounded (funnel-differentiated) adstock transformations, and interaction terms, demonstrates methodological soundness and practical applicability.

The key empirical result from the case study is the discovery of a statistically significant negative interaction elasticity (interference) between most advertising channels (in particular, lower-funnel formats such as Sponsored Products) and price promotions. This outcome identifies audience cannibalization and diminishing marginal returns that emerge when these instruments are applied concurrently.

Achieving the stated objectives enabled the formulation of practically relevant recommendations for marketers. The work empirically demonstrates that the intuitive maximum pressure strategy (simultaneous launch of all activities) is suboptimal. Optimizing aggregate ROI requires desynchronizing (staggering) marketing efforts: conducting advertising campaigns (especially in the upper and mid funnel) before peak promo periods rather than in parallel with them. The proposed simulation of the optimal mix (60–68% for advertising and 32–40% for promotions) provides managers with a data-based benchmark for budget allocation.

The study is intentionally scoped as an applied, case-based demonstration of an elasticity-focused MMM rather than a fully causal evaluation of marketing interventions. Accordingly, the empirical design is deliberately based on an aggregated weekly time series for a single retailer over a one-year window and operationalizes promotions as total discount volume and advertising as channel-level spends transformed via adstock, with interaction terms capturing contemporaneous joint effects. This scope is chosen to reflect the data granularity that is most commonly available to practitioners and to keep the framework readily implementable within standard analytics pipelines. For the same reason, the model focuses on within-retailer variation and does not attempt cross-market generalization, brand-level heterogeneity, or explicit modeling of nonlinear saturation and long-run structural dynamics beyond the geometric adstock specification. The results are therefore intended to support local, decision-relevant budget diagnostics and scenario comparisons in similar retail e-commerce settings, conditional on the observed planning regime and calendar environment. Acknowledging the limitations inherent in econometric modeling on aggregated data, this study delineates a trajectory for further development. The future of MMM lies in integration with Bayesian and causal methods, enabling a transition from measuring correlations to a genuinely causal understanding of market drivers.

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